Loading Rate Effects of High Damping Seismic Isolation Rubber Bearing on Earthquake Responses

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This paper investigates the effects of the loading rates of high damping seismic isolation rubber bearing on earthquake responses. For this purpose, a seismically isolated system is formulated by the Runge-Kutta numerical algorithm with nonlinear rate model of high damping rubber bearing to carry out the seismic time history response analysis. Results of the seismic time history response analyses using both the present parameter equations and Fujita's equations are compared with those of pseudo dynamic tests. From these results it is confirmed that the parameter equations of high damping rubber bearing should be obtained by tests with the loading rate equivalent to the isolation frequency as close as possible.

Key Words: Seismic Isolation, High Damping Rubber Bearing, Nonlinear Rate Model, Loading Rate, Parameter Equations

Nomenclature

A_s	: Shear area
$[C]$	: Damping matrix
F_1, F_2, F_u, F_m	: Variables of restoring forces
F_{eq}	: Equivalent excitation frequency
$F(x_r, \dot{x}_r)\|_{iso}$	: Restoring force of seismic isolation bearing
G_{eq}	: Equivalent shear modulus
H_{eq}	: Equivalent viscous damping constant
H_R	: Total height of rubber
$[K]$	: Stiffness matrix
K_0	: Equivalent stiffness
K_1, K_2	: Constants
$[M]$	: Mass matrix
M_t	: Total mass of superstructure
U	: ratio as F_u/F_m
$\ddot{x}_g$	: Input acceleration earthquake
X_m	: Maximum displacement
x_r	: Relative displacement vector
$y, \dot{y}$	: Transformation displacement and velocity vector
γ	: Shear strain

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1. Introduction

Recently, seismic isolation system has drawn worldwide attentions for its high performance of energy dissipation and period shift during earthquake events. Many researchers are eagerly trying to develop a design guideline of seismic isolation system applicable to many fields such as, building, apartments, nuclear power plant, and so forth.

First of all, for the design of seismically isolated system, it is very important to evaluate and model the seismic isolator characteristics as accurate as possible. The rubber bearing has complex non-linear characteristics in relationship between the restoring force and shear strain.

To make a mathematical model for the non-linear behavior of the laminated rubber bearing, many researchers use the equivalent spring-damper model, simple bi-linear model, modified bi-linear model, Ramberg-Osgood model, rate model, and so forth(Bhatti and Pister, 1981; Fujita, et al., 1990; Ohtori and Ishida, 1995; Koo and Lee, 1996). Among these mathematical models, it is known that the rate model gives more accurate results in the range of design shear strain(Fujita et al., 1990). Therefore, in this paper, the rate model is used for rubber bearing model and solved by using the Runge-Kutta numerical algorithm.

M. A. Bhatti used the basic rate model which requires the data such as yields force, yield deformation, a constant which controls the slop after yielding, and some material parameters obtained by experiments(Bhatti and Pister, 1981). In this paper, the modified rate model is used, which is controlled by three parameter equations ; the shear modulus, a ratio of restoring force at zero displacement to maximum restoring force, and the equivalent viscous damping(Fujita et al., 1990 ; Ohtori and Ishida, 1995).

In determination of the parameter equations by loading tests of the laminated rubber bearing, the loading delay time between the time interval of the input wave data and the actual loading in tests causes the variation of the mechanical characteristics such as shear stiffness and damping(Ohtori and Ishida, 1995.).

For convenience, the loading rate is defined as the ratio of the time interval of input motion to the loading time interval in test machine. The main purpose of this paper is to investigate the importance of these loading rates in developing the general design guideline for high damping seismic isolation rubber bearing model. For this purpose the seismic time history response analyses are performed for a single degree of freedom model of seismically isolated system which has a isolated frequency, 0.5 Hz using the present parameter equations which are in consideration of the loading rates and the Fujita's parameter equations(Fujita et al., 1990) which do not consider of the loading rates. And also these loading rate effects on earthquake responses are verified by the results of the pseudo dynamic tests.

2. Formulations of Seismically Isolated System

2.1 Formulation using the Runge-Kutta numerical algorithm

In general, the governing equation of motion for a seismically isolated system, which uses the nonlinear seismic isolators, can be expressed with simple lumped mass, damping, and stiffness matrix of superstructure as follows ;

$$[M]\{\ddot{x}_r\}+[C]\{\dot{x}_r\}+[K]\{x_r\} +\{F(x_r,\ \dot{x}_r)|_{iso}\}=-[M]\{\ddot{x}_g\} \qquad (1)$$

In Eq. (1), $\{x_r\}$ is relative displacement for input motion and $\{\ddot{x}_g\}$ is input acceleration earthquake, and $\{F(x_r,\ \dot{x}_r)|_{iso}\}$ indicates the nonlinear restoring force vector of seismic isolation rubber bearing which is a function of displacement and velocity responses of isolator. To solve the above equation this paper introduces the Runge-Kutta numerical algorithm using the transformation vectors as follows ;

$$y=\begin{Bmatrix} x_r \\ \dot{x}_r \end{Bmatrix},\quad \dot{y}=\begin{Bmatrix} \dot{x}_r \\ \ddot{x}_r \end{Bmatrix} \qquad (2,\ 3)$$

From Eq. (1) the $\{\ddot{x}_r\}$ can be expressed as follows ;

$$\{\ddot{x}_r\}=-[M]^{-1}([C]\{\dot{x}_r\}+[K]\{x_r\}+[M]\{\ddot{x}_g\} +\{F(x_r,\ \dot{x}_r)|_{iso}\}) \qquad (4)$$

After substituting Eq. (4) to (3), the second order differential Eq. of (1) is transformed to the

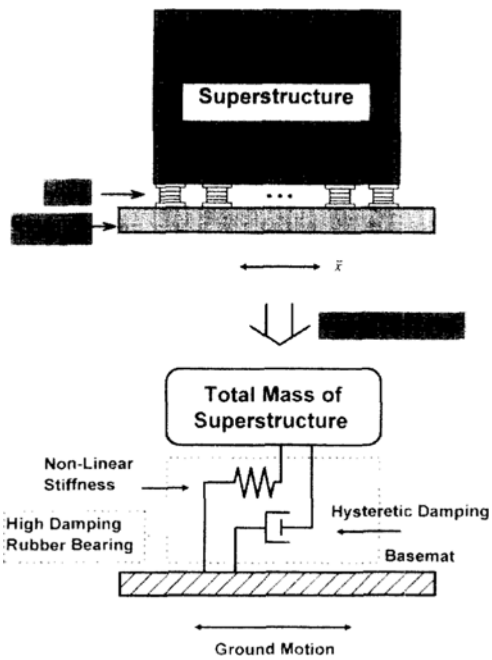


Fig. 1 One dimensional model of isolated system.

first order differential equation as follows ;

$$\{\dot{y}\}=\begin{bmatrix} 0 & I \\ -[M]^{-1}[K] & -[M]^{-1}[C] \end{bmatrix}\{y\} + \begin{Bmatrix} 0 \\ -\{\ddot{x}_g\}-[M]^{-1}\{F(x_r,\ \dot{x}_r)|_{iso}\} \end{Bmatrix} \tag{5}$$

In this paper, we represent seismically isolated structure with a single degree of freedom system as shown in Fig. 1. In this case, the Eq. (5) can be simply expressed as follows ;

$$\{\dot{y}\}=-\begin{Bmatrix} 0 \\ \ddot{x}_g+F(x_r,\ \dot{x}_r)/M_t \end{Bmatrix} \tag{6}$$

In Eq. (6), M_t is total mass of superstructure with Kg unit.

2.2 Nonlinear rate model of seismic isolation rubber bearing

For the mathematical model of seismic isolation bearing, we use the modified nonlinear rate model(Fujita et al., 1990 ; Ohtori and Ishida, 1995). Figure 2 shows the basic concept of the nonlinear rate model. With this model, the total restoring force of rubber bearing in Eq. (6) can be expressed as

$$F=F_1+F_2 \tag{7}$$

$$F_1=K_2 \cdot x_r \tag{8}$$

$$\dot{F}_2=(K_1-K_2)\cdot \dot{x}_r \cdot \left\{1.0-sgn(\dot{x}_r)\left(\frac{F_2}{F_u}\right)^n\right\} \tag{9}$$

where the dot on parameters indicates the time derivative, sgn ( ) means the sign change of ( ), i.e. $sgn(\dot{x}_r)=|\dot{x}_r|/\dot{x}_r$ and is the restoring force at zero displacement during maximum hysteresis loop responses as shown in Fig. 2. In Eq. (9), n is a material parameter, taken as an odd integer which controls the sharpness of transient from the elastic to inelastic region. As n goes to the infinite, the Rate Model approaches a bilinear model (Bhatti and Pister, 1981). In this paper, approximately $n=1$ is used for the sharpness of rate model(Fujita et al., 1990). In Eqs. (8) and (9), the K_1 and K_2 are calculated as functions of maximum shear strain γ as follows ;

$$K_1=[1.0-U(\gamma)+U(\gamma)/S]\cdot K_0 \tag{10}$$

$$K_2=[1.0-U(\gamma)]\cdot K_0 \tag{11}$$

In Eqs. (10) and (11), U is the ratio of F_u to F_m shown in Fig. 2 and K_0 is the equivalent stiffness calculated by

$$K_0=\frac{G_{eq}(\gamma)\cdot A_x}{H_R} \tag{12}$$

where H_R is the total height of rubber. In Eq. (10), S is the value that satisfies the following equations,

$$S\cdot\frac{e^{2/S}-1}{e^{2/S}+1}=\frac{U(\gamma)-Q(\gamma)}{U(\gamma)} \tag{13}$$

$$Q(\gamma)=\pi\cdot H_{eq}(\gamma)/2 \tag{14}$$

where e indicates the exponential function and H_{eq} is a equivalent viscous damping constant. In Eq. (13), some numerical iterations are required to obtain the S. For large shear strain over about 150%, the Eq. (13) may diverge. This makes the Rate Model unfit for applications with large shear strain of high damping rubber bearing.

In above equations, γ is the shear strain at the maximum displacement of X_m shown in Fig. 2 obtained by the following equation,

$$\gamma=X_m/H_R \tag{15}$$

The parameter equations such as $G_{eq}(\gamma)$, $U(\gamma)$, and $Q(\gamma)$ should be obtained by tests of rubber bearing. These equations are described in the following chapter.

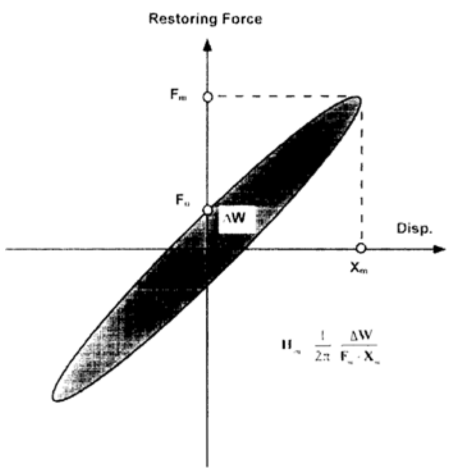
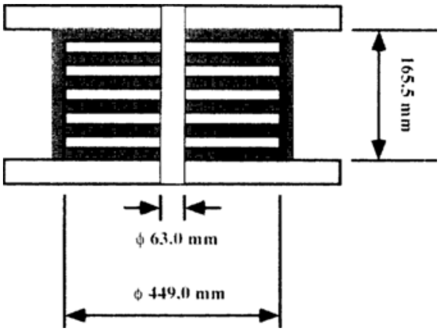


Fig. 2 Basic concept of non-linear rate model.

3. Parameter Equations

In this paper, two types of high damping seis-



Rubber : 31 x 2.5 mm
Steel : 30 x 1.6 mm
Similarity Ratio : 3.16

Fig. 3 Layout dimension of high damping rubber bearing.

mic isolation rubber bearing are used. One is the high damping rubber bearing used in this paper and the other one is that used by T. Fujita.

The layout dimensions of high damping rubber bearing used in this paper is shown in Fig. 3. This is a reduced model with similarity ratio of 3.16. The seismic isolation frequency used in this paper is $0.5Hz$. To achieve this isolation frequency, the total mass of superstructure is assumed as 158.274 Kg. From these conditions, three sets of parameter equations are obtained by loading tests with three equivalent excitation frequencies of $0.01Hz$, $0.25Hz$, and $0.5Hz$. These frequencies are determined by the loading rates in test machine as follows ;

$$\text{Loading Rate} = \frac{\textit{Time Interval of Input Motion } \Delta t}{\textit{Loading time Interval}} \tag{16}$$

Equivalent Excitation Frequency $(F_{eq}) \approx$ *Isolation Frequency* x *Loading Rate.* (17)

In this paper, the equivalent excitation frequencies, of $0.01Hz$, $0.25Hz$, and $0.5Hz$ are corresponding to the loading rates, 1/50, 1/2, and 1/1, respectively.

The parameter equations obtained by test results in consideration of the loading rates are as follows (Ohtori and Ishida, 1995) ;

For $F_{eq}=0.01Hz$:

$$G_{eq}(\gamma)=0.01323-0.01435\gamma+0.009045\gamma^2-0.001854\gamma^3 \tag{18}$$

$$U(\gamma)=0.2773-0.1703\gamma+0.1507\gamma^2-0.04903\gamma^3 \tag{19}$$

$$Q(\gamma)=0.2394-0.1494\gamma+0.1606\gamma^2-0.05213\gamma^3 \tag{20}$$

For $F_{eq}=0.25Hz$:

$$G_{eq}(\gamma)=0.01661-0.01832\gamma+0.01114\gamma^2-0.002238\gamma^3 \tag{21}$$

$$U(\gamma)=0.2804-0.06308\gamma+0.1149\gamma^2-0.0516\gamma^3 \tag{22}$$

$$Q(\gamma)=0.247-0.0493\gamma+0.1049\gamma^2-0.04481\gamma^3 \tag{23}$$

For $F_{eq}=0.50Hz$:

$$G_{eq}(\gamma)=0.01977-0.02191\gamma+0.01306\gamma^2-0.002578\gamma^3 \tag{24}$$

$$U(\gamma)=0.2775-0.04915\gamma+0.0959\gamma^2-0.04501\gamma^3 \tag{25}$$

$$Q(\gamma)=0.2568-0.08893\gamma+0.1469\gamma^2-0.05686\gamma^3 \tag{26}$$

In Eqs. (18) ~ (26), $U(\gamma)$ and $Q(\gamma)$ are non-dimensional parameters and $G_{eq}(\gamma)$ has a unit of ton/cm^2.

Another parameter equations considered in this paper is the T. Fujita's results (Fujita etal., 1990). The rubber bearing used in his paper comprise of 23 rubber layers of 6 mm thickness and 450 mm diameter bonded to steel plates. He obtained the parameter equations using 0.2 Hz equivalent excitation frequency for two types of rubber bearing which have a difference in hardness of rubber as follows ;

For low hardness rubber (IRHD=50) : $0 \le \gamma \le 0.7$ region

$$K_0(\gamma)=2.03-4.97\gamma+6.73\gamma^2-3.34\gamma^3 \tag{27}$$

$$U(\gamma)=0.363-0.231\gamma+0.0357\gamma^2+0.0609\gamma^3 \tag{28}$$

$$Q(\gamma)=0.268-0.109\gamma+0.07\gamma^2-0.0186\gamma^3 \tag{29}$$

For low hardness rubber (IRHD=50) : $0.7 \le \gamma \le 1.4$ region

$$K_0(\gamma)=1.08-0.548\gamma-0.0376\gamma^2+0.107\gamma^3 \tag{30}$$

$$U(\gamma)=0.475-0.671\gamma+0.606\gamma^2-0.181\gamma^3 \tag{31}$$

$$Q(\gamma)=0.261-0.0829\gamma+0.0414\gamma^2-0.00952\gamma^3 \tag{32}$$

For high hardness rubber (IRHD=60) : $0 \le \gamma \le 0.$

7 region

$$K_0(\gamma)=3.02-7.47\gamma+10.1\gamma^2-5.03\gamma^3 \quad (33)$$
$$U(\gamma)=0.448-0.561\gamma+0.873\gamma^2-0.517\gamma^3 \quad (34)$$
$$Q(\gamma)=0.282-0.0168\gamma-0.0537\gamma^2+0.0487\gamma^3 \quad (35)$$

For high hardness rubber (IRHD=60) : $0.7\leq\gamma\leq 1.4$ region

$$K_0(\gamma)=2.36-3.40\gamma+2.81\gamma^2-0.0748\gamma^3 \quad (36)$$
$$U(\gamma)=0.363-0.231\gamma+0.036\gamma^2+0.061\gamma^3 \quad (37)$$
$$Q(\gamma)=0.261-0.083\gamma+0.041\gamma^2-0.010\gamma^3 \quad (38)$$

In Eqs. (27) ~ (38), $K_0(\gamma)$ has a unit, MN/m. In T. Fujita's research, he used the parameter equation, $K_0(\gamma)$ instead of $G_{eq}(\gamma)$. In this case, the value of $K_0(\gamma)$ depends on the size of rubber bearing. Therefore, for the general applications, $K_0(\gamma)$ can be easily transformed to the equivalent shear modulus, $G_{eq}(\gamma)$ by using the Eq. (12).

4. Analysis and Discussions of Loading Rate Effects

For the input motion, the synthetic earthquake data, shown in Fig. 4, which is made by the measurement data in Mexico, 1985 and tentative design response spectrum (Ishida etal., 1989), is used. This earthquake is often used in design and evaluation of seismically isolated system due to its long period components. The maximum peak acceleration of the Mexico earthquake is actually 316 gal but in this paper 100 gal is used, which is consistent with the similarity ratio of 3.16.

The total analysis time is 50 seconds and the analysis time interval of input motion is 0.005 sec.

The pseudo dynamic tests (Ohtori and Ishida, 1995) are carried out to verify the loading rate effects on earthquake responses. Figure 5 shows the basic concepts of pseudo dynamic tests. In the tests, we get the restoring forces from test machine and solve the equation of motion with this in computer and put the displacement response to test machine for excitations. Table 1 shows the capacity specification of the test machine.

Figures 6 and 7 show the responses of hysteresis loop obtained by analyses and pseudo dynamic tests for each loading rate, respectively. The shapes of hysteresis loop of analyses are in very good agreement with that of pseudo dynamic tests. From the results, we can see that the stiffness increases while the maximum displacements decrease as loading rates increase. Figure 8 shows the isolation frequency variations caused by loading rate effects. These results can be reconfirmed by displacement time history responses as shown in Fig. 9. The response wave forms obtained by analyses are good agreement with those of pseudo dynamic tests. In these results, the maximum displacement is reduced as loading rate increases, which strongly suggests that the loading rates affect the characteristics of the high damping rubber bearing.

Figure 10 shows the time history of the displacement response obtained by using the Fujita's parameter equations. As mentioned in previously chapter, his parameter equations are for 0.2 Hz excitation tests. Therefore, the results

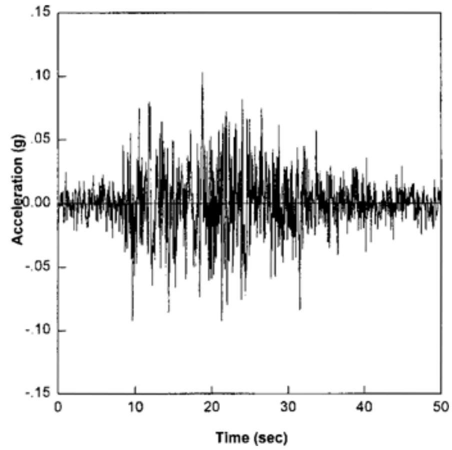


Fig. 4 Synthetic design earthquake from 1985 Mexico earthquake.

Table 1 Capacity specification of test machine.

	Horizontal actuator	Verical actuator
Load	±50tons	±100tons
Displacement	±300mm	±300mm
velocity	±70cm/sec	±35cm/sec
Control system	Displacement control or Load control	
Waveform	Arbitrary waveform by external input	
Frequency range	0Hz~30Hz	

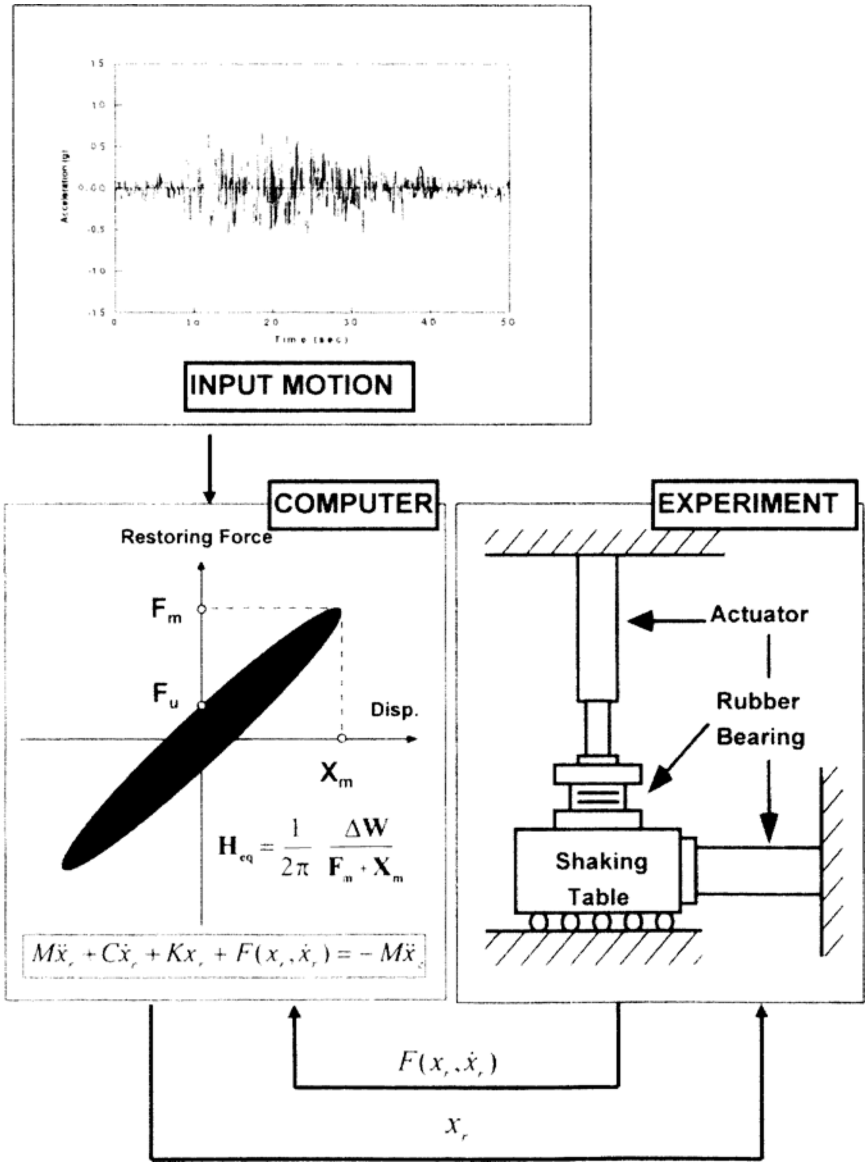


Fig. 5 Basic concept of pseudo dynamic test.

obtained by Fujita's parameter equations are very similar to the results of 0.25 Hz excitation of this study. But when considering the isolated system, which will be dominantly excited by the excitation frequency of 0.5 Hz, the Fujita's parameter equations may result in larger displacement responses than the actual responses. It is necessary to consider the loading rates when constructing the parameter equations of the rubber bearing by tests.

Figure 11 shows the loading rate effects in maximum peak displacement responses. From Fig. 11, it is shown that the loading rates significantly affect the maximum displacement responses. The differences of maximum displacement responses between 0.01 Hz and 0.5 Hz are 25% and 33% in analysis and experiments, respectively. Figure 12 shows the comparison of maxi-

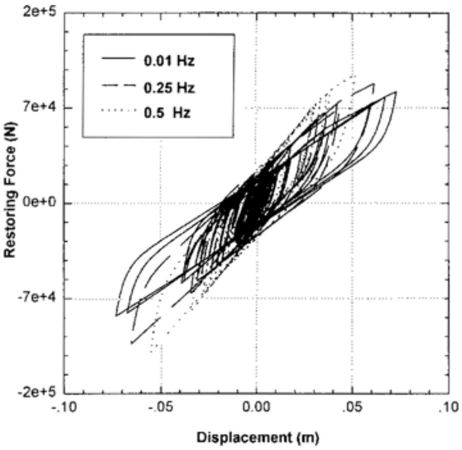


Fig. 6 Results of hysteresis loop by analysis.

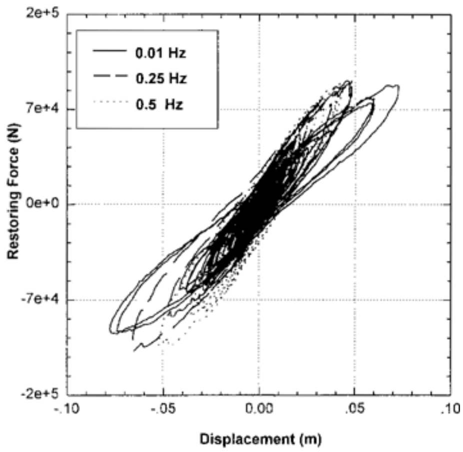


Fig. 7 Results of hysteresis loop by pseudo dynamic test.

mum peak acceleration responses. As the loading rates increase, the maximum peak acceleration increase. This means that the parameter equations of high damping rubber bearing obtained by lower excitation frequencies than the isolation frequency may result in under estimated maximum peak acceleration responses. In actual seismic isolation system which has 0.5 Hz isolation frequency, the rubber bearing will be dominantly excited by the component of 0.5 Hz. Therefore, in Figs. 11 and 12, the peak value obtained by the parameter equations of 0.5 Hz excitation can give much more accurate results in actual earthquake events. This means that the parameter equations should be obtained by tests with the loading rate

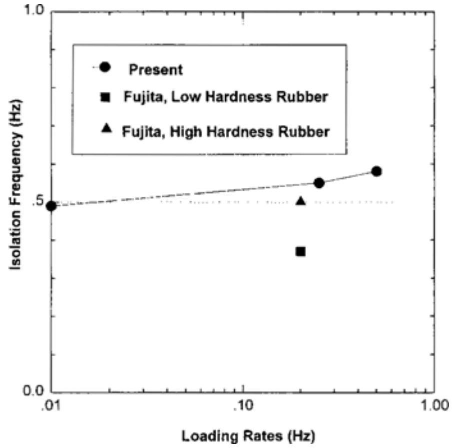


Fig. 8 Variation of isolation frequencies by loading rates.

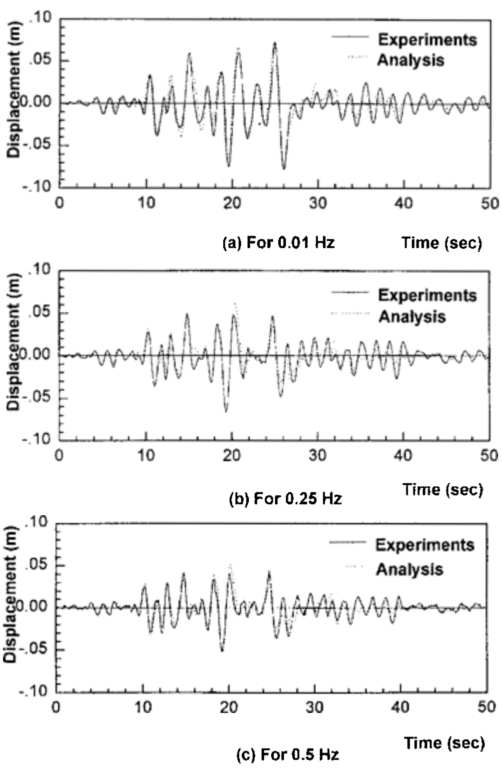


Fig. 9 Results of displacement time history responses.

equivalent to the isolation frequency as close as possible.

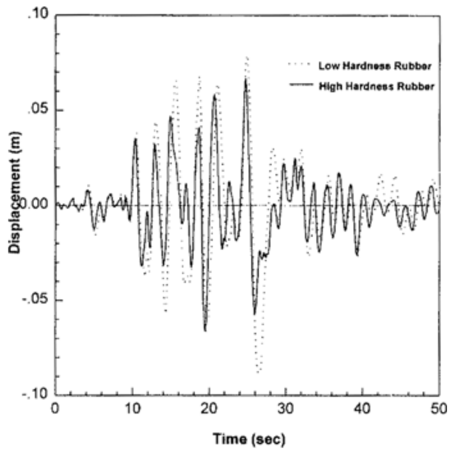


Fig. 10 Results of displacement time history responses using Fujita's parameter equations.

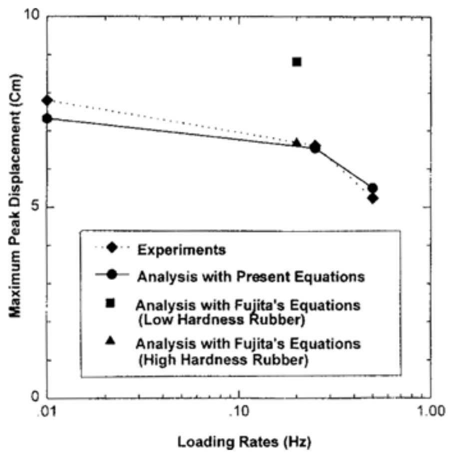


Fig. 11 Variation of maximum peak displacement by loading rates.

5. Conclusions

For the evaluation of seismic isolation rubber bearing, there can be two tests such as quasi static test and the dynamic test. As mentioned in above results, the slow loading rates result in lower stiffness and smaller hysteresis damping than the fast loading rates. Therefore, the test results which are not performed with the loading rate closed to the isolation frequency can result in wrong evaluation of a characteristics of high damping rubber bearing for actual earthquake events.

In conclusion, the parameter equations of high

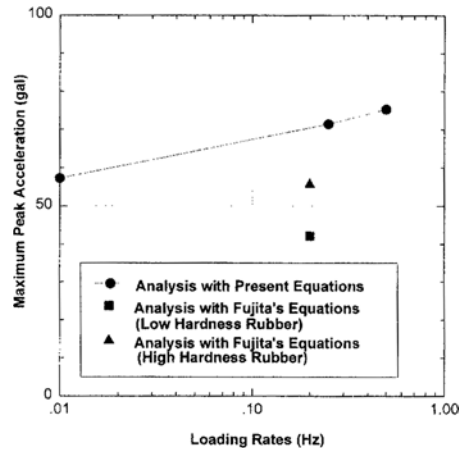


Fig. 12 Variation of maximum peak acceleration by loading rates.

damping rubber bearing for a mathematical model should be obtained by loading tests with the loading rate equivalent to the isolation frequency for more accurate results in design and analysis of seismically isolated system.

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